Theorem :- Establishes generalised form of De Morgan's law.

Statement :- If $\{A_i : i \in I\}$ be an indexed family of subsets of the ~~inter~~ Universal set Ω then

(i) complement of the Union of an indexed family of sets is the intersection of their complements.

i.e. $\left(\bigcup_{i \in I} A_i\right)' = \bigcap_{i \in I} A_i'$

(ii) The complement of the intersection of an indexed family of sets is the Union of their complements.

i.e. $\left(\bigcap_{i \in I} A_i\right)' = \bigcup_{i \in I} A_i'$

Proof :- (i) Let a be any element of Ω

$\Rightarrow a \in \left(\bigcup_{i \in I} A_i\right)' \Leftrightarrow a \notin \bigcup_{i=I} A_i$

$\Rightarrow a \notin$ any $A_i \Leftrightarrow a \notin$ each A_i'

$\Leftrightarrow a \in \bigcap_{i=I} A_i'$

$\therefore \left(\bigcup_{i \in I} A_i\right)' \subseteq \bigcap_{i \in I} A_i'$

And $\bigcap_{i \in I} A_i' \subseteq \left(\bigcup_{i \in I} A_i\right)'$

Thus $\bigcup_{i \in I} A_i' = \bigcap_{i \in I} A_i'$

(ii) Let a be any element of Ω

$\Rightarrow a \in \left(\bigcap_{i \in I} A_i\right)' \Leftrightarrow a \notin \bigcap_{i \in I} A_i$

$\Leftrightarrow a \notin A_i$ for at least one $i \in I$

$\Leftrightarrow a \in A_i'$ for at least one $i \in I \Leftrightarrow$

$\therefore \left(\bigcap_{i \in I} A_i\right)' \subseteq \bigcup_{i \in I} A_i'$

and $\bigcup_{i \in I} A_i' \subseteq \left(\bigcap_{i \in I} A_i\right)'$

Thus $\left(\bigcap_{i \in I} A_i\right)' = \bigcup_{i \in I} A_i'$